Achieving Spectrum Efficiency Through Signal Design for Ultra Wide Band Sensor Networks

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Abstract - In this paper, a novel approach is presented for ultra wide band signal design. The bottom up approach designs the signals that utilize the ultra wide band spectrum efficiently. 1st-6th derivative Gaussian pulses are linearly combined using a particle swarm optimization algorithm to form one single pulse. A binary PSO determines the order of the derivative of the pulses that are combined. The continuous PSO determines the time duration and amplitudes for different pulses in the composite pulse. The power spectral density (PSD) of the resultant pulse conforms to the FCC spectral mask and effectively exploits the allowable bandwidth and power. The particle swarm optimization algorithm achieves multiple orthogonal pulses. The newly designed pulses achieve higher spectral efficiencies which is shown theoretically and in simulations. The proposed method presents a flexible and effective way for generating UWB pulses that satisfy the FCC mask. The method can be generalized to design UWB pulses for any given spectral mask.

I. INTRODUCTION

UWB technology is a promising solution for short-range, high-speed, wireless communication systems. Instead of using carrier frequencies, UWB systems transmit information using trains of short time duration pulses, which spread the energy from 0Hz to a few GHz. Due to its ultra wide bandwidth, UWB devices interfere with other narrow band communication systems. The Federal Communications Commission (FCC) released the regulations in February 2002 that set the power emission limits for all UWB devices [1]. This limits the interference caused by these devices while allowing them to occupy a huge band. Critical to the performance of a UWB system performance is its capability to maximize the power that it can propagate within this mask. Research in UWB systems in this decade has been focussing on signal design to maximize the resultant power spectral efficiency since the spectrum is a limited resource.

Different signal design techniques are studied. Gaussian 2nd derivative pulses are investigated in [2], but they are not flexible to conform to the FCC spectral masks and are filtered. A pulse set based on modified Hermite polynomials (HP) are orthogonal to one another [3], but frequency shifting is necessary for the HP pulses of order 0 and 1 to meet the FCC spectral masks. Higher order HP pulses are susceptible to timing jitter and noise, and need bandpass filters to fit their PSD into the FCC masks. Pulses designed utilizing the ideas of prolate spheroidal (PS) functions satisfy the FCC mask [4], but they do not effectively exploit the allowable bandwidth and power. The orthogonality of both HP pulses and PS pulses is only preserved when each user is assigned a unique pulse in a perfectly synchronous, multiuser, UWB system without considering the channel distorting effect and antenna response characterization, rendering it impractical. Pulses based on a linear combination of a set of base waveforms obtained by differentiation of the Gaussian pulse was introduced in [4], [6]. However, the strategies for selecting the combination and coefficients is random, and the algorithm designing the solutions is based on the huge number of trial and error iterations. This makes the results neither determined nor reproducible. In addition, pulses generated 1st to 15th Gaussian derivative pulses are combined. Hardware complexity increases as the order of the derivative increases.

To accomplish the pulse design, we propose a novel design method using Particle Swarm Optimization (PSO) algorithm. A binary PSO is used to select the pulses that form the composite pulse. A continuous PSO is used to obtain the parameters (time duration and amplitude) for each of the selected pulses. We compare the results achieved by the PSO to an exhaustive approach employed before. The new composite pulse generated conforms to the FCC indoor mask and maximizes the power. Multiple orthogonal pulses are derived from this PSO generated composite signal. This enables us to manage our network according to different system requirements by switching between signals (pulses) that sensors use.

This paper is organized as follows. In Section II, we introduce the base waveforms - Gaussian derivative pulses. For different baseband modulation schemes, the resultant PSD is proportional to the PSD of individual UWB pulse used. In
Section III, a new pulse shape design method is proposed. In Section IV, the UWB communication simulation system is presented. Results and conclusions are presented in Section V.

II. GAUSSIAN DERIVATIVE PULSES AND THEIR PSD
A Gaussian pulse is considered in this paper. The Gaussian pulse in our design has the form [7]

\[ s(t) = \frac{A}{\sqrt{2\pi \sigma^2}} e^{-\frac{t^2}{2\sigma^2}} \frac{-2\pi^2}{\alpha} = \sqrt{\frac{2}{\alpha}} A e^{-\frac{t^2}{\alpha^2}} \]  

(1)

where \( A \) is the amplitude of the pulse, and \( \alpha = \sqrt{4\pi \sigma^2} \) represents a bandwidth scaling factor. The antenna has an effect of differentiating the time waveform presented to it. Since the pulse passes through the transmitter and receiver antennas, it is differentiated twice at the output of the receiver antenna. The pulse is then given by

\[ s(t) = A(-\frac{t^2}{\sqrt{2\pi \sigma^5}} - \frac{1}{\sqrt{2\pi \sigma^3}}) e^{-\frac{t^2}{2\sigma^2}} \frac{2\pi^2}{\alpha^2} \]  

(2)

The PSD of a waveform can be expressed as

\[ PSD(f) = |S(f)|^2 = \left| \int_{-\frac{1}{2}T_D}^{\frac{1}{2}T_D} s(t) e^{-j2\pi ft} dt \right|^2 \]  

(3)

and total power is calculated using

\[ \text{power} = \int_{f} |S(f)|^2 df \]  

(4)

In Figure 2, PSD for Gaussian 1st - 6th derivative pulses show the shift of power to higher frequencies as the order of derivative increases, for arbitrary \((A, \alpha)\) values. The FCC mask that the signals should comply is shown in blue.

<table>
<thead>
<tr>
<th>Gaussian Pulses</th>
<th>EXPRESSIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Derivative</td>
<td>( s_1(t) = 4\pi t e^{-\frac{t^2}{\alpha^2}} )</td>
</tr>
<tr>
<td>2nd Derivative</td>
<td>( s_2(t) = (-4\pi(-\alpha^2 + 4\pi^2) e^{-\frac{2\pi^2}{\alpha^2}}) / \alpha^4 )</td>
</tr>
<tr>
<td>3rd Derivative</td>
<td>( s_3(t) = (16\pi^2 t(-3\alpha^2 + 4\pi^2) e^{-\frac{2\pi^2}{\alpha^2}}) / \alpha^6 )</td>
</tr>
<tr>
<td>4th Derivative</td>
<td>( s_4(t) = (-16\pi^2(3\alpha^2 - 24\pi^2 \alpha^4 + 16\pi^4 t^4) e^{-\frac{2\pi^2}{\alpha^2}}) / \alpha^8 )</td>
</tr>
<tr>
<td>5th Derivative</td>
<td>( s_5(t) = (64\pi^2 t(5\alpha^4 - 40\pi^2 \alpha^2 + 16\pi^4 t^4) e^{-\frac{2\pi^2}{\alpha^2}}) / \alpha^{10} )</td>
</tr>
<tr>
<td>6th Derivative</td>
<td>( s_6(t) = (-64\pi^2 t(-15\alpha^2 + 180\pi^2 \alpha^4 - \frac{2\pi^2}{\alpha^2}) / 240\pi^2 t^4 \alpha^2 + 64\pi^4 t^6)) / \alpha^{12} )</td>
</tr>
</tbody>
</table>

Figure 1. Individually Optimized Pulses and their PSD. 1st to 6th Derivative of Gaussian pulses are shown.
By choosing the right order of the derivative to combine and suitable \((A, \alpha)\) values, a pulse that can fit the FCC mask can be found. In our design we use Gaussian 1-6th derivatives.

**TABLE II. INDIVIDUALLY OPTIMIZED PULSES AND THEIR CORRESPONDING POWER VALUES**

<table>
<thead>
<tr>
<th>Pulse</th>
<th>(\alpha) (ns)</th>
<th>(A(v))</th>
<th>Power (\mu W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Derivative</td>
<td>1.995e-9</td>
<td>0.832</td>
<td>24.87</td>
</tr>
<tr>
<td>2nd Derivative</td>
<td>1e-10</td>
<td>4.216</td>
<td>18.718</td>
</tr>
<tr>
<td>3rd Derivative</td>
<td>1.375e-10</td>
<td>6.16</td>
<td>63.938</td>
</tr>
<tr>
<td>4th Derivative</td>
<td>1.625e-10</td>
<td>11.52</td>
<td>209.79</td>
</tr>
<tr>
<td>5th Derivative</td>
<td>2.125e-10</td>
<td>9.88</td>
<td>243.77</td>
</tr>
<tr>
<td>6th Derivative</td>
<td>2.5e-10</td>
<td>8</td>
<td>156.37</td>
</tr>
</tbody>
</table>

The UWB pulse shape design involves selection of pulses from 1st to 6th derivative of the Gaussian pulse. Let us define a 6 bit binary vector given by

\[
\{b_1, b_2, \ldots, b_6\}
\]

where, \(b_i = 1\) implies that \(i^{th}\) derivative is used. There are two parameters that need to be designed for each selected pulse. To simultaneously achieve the pulse selection and the parameter values for the selected pulses, we define a vector of continuous values of length 12. The vector is decoded as follows. The first 6 give the \(\alpha\) values, the next 6 give the \(A\) values for the six pulses. The continuous vector is denoted by

\[
c = \{a_1, a_2, \ldots, a_6, a_1, a_2, \ldots, a_6\}
\]

where, \([a_i, a_i]\) give the time duration and the amplitude for the \(i^{th}\) derivative.

The final pulse from this is derived using

\[
s(t) = \sum_{i=1}^{6} b_i \times s_i(t, c_i, c_{i+6})
\]

The objective of the algorithm is to maximize (4) without violating the FCC mask. The PSO formulae define each particle as a potential solution to a problem in a D-dimensional space. Hence the \(i^{th}\) particle represented as \(X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})\), where ‘D’ is the dimension number. There are total of 18 dimensions in this problem based on (5) and (6) where x’s in the particle replace the b’s and c’s respectively. Each particle also maintains a memory (pbest) of its previous best position, \(P_i = (p_{i1}, p_{i2}, p_{i3}, \ldots, p_{iD})\) and a velocity along each dimension represented as \(V_i = (v_{i1}, v_{i2}, v_{i3}, \ldots, v_{iD})\). Given the particle the signal is constructed using (7). The PSD of the signal is calculated using (3). The performance of this particle is evaluated using the following conditional expression:

\[
F = \begin{cases} 
\text{power} & \text{if } PSD(f) \leq \text{Mask}(f) \\
0 & \text{otherwise}
\end{cases}
\]

where power is given by (4). (8) is one of the penalty functions available to handle the constraints on an optimization problem [9]. The step nature of the above objective function induces discontinuity limiting the ability to design a gradient based approach for optimization. In each generation, the pbest vector of the particle with the best fitness in the local neighborhood, designated as gbest, and the pbest vector of the current particle are combined to adjust the velocity along each dimension given by

\[
v_{id}^{(t+1)} = \omega \times v_{id}^{(t)} + \psi_1 \times (p_{id} - X_{id}) + \psi_2 \times (p_{gd} - X_{id})
\]

The portion of the adjustment to the velocity influenced by the individual’s own pbest position is considered as the cognition component, and the portion influenced by gbest is...
the social component. Constants $\psi_1$ and $\psi_2$ determine the relative influence of the social and the cognition components, and are often both set to the same value to give each component (the cognition and the social learning rates) equal weight. $V_{\text{max}}$, is often used to limit the velocities of the particles and improve the resolution of the search space.

The velocity is then used to compute a new position for the particle. The position update for the continuous part of the particle is given by

$$X_{id}^{(t+1)} = X_{id}^{(t)} + V_{id}^{(t+1)}$$  \hspace{1cm} (10)

For position update of the binary component of the particle, first the velocity is transformed into a $[0, 1]$ interval using the sigmoid function given by

$$\text{sigmoid}(V_{id}) = \frac{1}{1 + e^{-V_{id}}}$$  \hspace{1cm} (11)

where $V_{id}$ is the velocity of the $i$th particle’s $d$th dimension.

A random number is generated using a uniform distribution, which is compared to the value generated from the sigmoid function, and a decision is made about the $X_{id}^{(t+1)}$ from

$$X_{id} = u(\text{sigmoid}_{id} - U[0, 1])$$  \hspace{1cm} (12)

$u$ is a unit step function. The decision regarding $X_{id}^{(t+1)}$ is probabilistic.

**Algorithm PSO:**

For $t=1$ to the max. bound of the number on generations,

For $i=1$ to the population size,

For $d=1$ to the problem dimensionality,

Apply the velocity update equation:

$$V_{id}^{(t+1)} = \omega V_{id}^{(t)} + \psi_1 (p_{id} - X_{id}) + \psi_2 (p_{gd} - X_{id})$$

where $p_i$ is the best position visited so far by $X_i$,

and $p_g$ is the best position visited so far by any particle;

Limit magnitude:

$$V_{id}^{(t+1)} = \min(V_{\text{max}}, \max(-V_{\text{max}}, V_{id}^{(t+1)}))$$

Update Position:

$$X_{id}^{(t+1)} = \min(\text{Max}_{\text{up}}, \max(-\text{Min}_{\text{down}}, X_{id}^{(t)} + V_{id}^{(t+1)}))$$

End-for-d;

Compute fitness of $X_{id}^{(t+1)}$;

If needed, update historical information regarding $p_i$ and $p_g$;

End-for-i;

Terminate if $p_g$ meets problem requirements;

End-for-t;

End-algorithm.

Figure 3: Pseudo Code of Particle Swarm Optimization Algorithm

**IV. UWB COMMUNICATION SYSTEM**

UWB signal radiates a train of pulses that are very short in time (typically a few nano seconds). The pulses are modulated by the information data symbols using modulation schemes that shape the spectrum of the generated signal.

For a binary TH-BPSK (time-hopping binary phase shift keying) UWB system, the $k$th user’s signal conveying $i$th bit can be represented as

$$s_p^{(k)}(t,i) = \sum_{j=N_s}^{i+1} d_j^{(k)} s(t - jT_f - \epsilon_j^{(1)} T_c)$$ \hspace{1cm} (13)

and a typical TH-PPM (time-hopping pulse position modulation) UWB signal takes the form

$$s_p^{(k)}(t,i) = \sum_{j=N_s}^{i+1} s(t - jT_f - \epsilon_j^{(1)} T_c - \epsilon d_i^{(k)})$$  \hspace{1cm} (14)

where $p(t)$ represents the transmitted pulse that we get at the output of transmitter antenna; $N_s$ is the number of pulses to represent one data bit; $T_f$ and $T_c$ are the frame and the chip duration respectively. The bit duration is $T_b = N_s T_f$;

$\{c_j^{(k)}\}$ is the distinct TH PN codes sequence of $k$th signal,

and $c_j^{(k)} \in [1, N_s]$; $\{d_i^{(k)}\}$ is the binary data sequence. In antipodal TH-BPSK UWB systems, we encode information using $d_i^{(k)} \in \{-1,1\}$. In TH-PPM UWB systems, we encode using $d_i^{(k)} \in \{0,1\}$, $\epsilon$ is the time offset of binary PPM. If $T_p$ is the pulse width, the limits $T_p + \delta < T_c$ and $N_s T_c \leq T_f$ are assumed. Note that a frame is divided into many chips, and the pulse generated from the $k$th user occupies only one of those. In order to get the PSD of a TH-PPM signal, we need to relax the hypothesis of an inconsequential effect of the time shift $\epsilon$, because the PSD of a PPM signal is hard to evaluate due to the non-linear nature of PPM modulation.

Let us define a new signal $v(t)$ as

$$v(t) = \sum_{j=0}^{N_s} s(t - jT_f - \theta_j)$$ \hspace{1cm} (15)

The power spectrum of $v(t)$ is

$$S_v(f) = \sum_{n=-\infty}^{\infty} e^{-j2\pi f (nN_s + \theta_n)}$$ \hspace{1cm} (16)

where $S(f)$ is the Fourier transform of $s(t)$.

Then the original UWB TH-PPM signal turns out to be

$$s_p^{(k)}(t,i) = \sum_{j=N_s}^{i+1} v(t - jT_f - \epsilon d_i^{(k)})$$ \hspace{1cm} (17)

If we can assume that $d_i^{(k)}$ is equiprobable to be 0 and 1, the transmitted signal PSD [7] is
\[
S_{\text{PPM}}(f) = \frac{|S(f)|^2}{T_b} \left[ 1 - |W(f)|^2 + \frac{|W(f)|^2}{T_b} \sum_{n=1}^{N_c} \delta (f - \frac{n}{T_b}) \right]
\]

where
\[
|W(f)|^2 = \frac{1}{2} (1 + \cos(2\pi f T))
\]

The PSD of an antipodal TH-BPSK UWB signal, \(S_{\text{BPSK}}(f)\) [7], is
\[
S_{\text{BPSK}}(f) = \frac{\sigma_d^2}{T_b} |S(f)|^2 + \frac{\mu_d^2}{T_b} \sum_{n=1}^{N_c} 1_\{\frac{n}{T_b}\} \delta (f - \frac{n}{T_b})
\]

where \(\sigma_d^2\) and \(\mu_d^2\) are respectively the variance and the mean of \(d^{(k)}\) sequences. It can be seen that the PSD of TH-PPM and TH-BPSK signal both have continuous and discrete components. However, the discrete spectral spikes vanish if the information sequences have zero mean. We assume this is true. The global effect of the time shift factor \(\varepsilon\) in TH-PPM signal on the severity of interference from UWB signal to co-existing systems is very small. Hence the PSD of both TH-PPM and TH-BPSK signal is to some extent linearly proportional to the PSD of the individual pulse. Hence for pulse design purposes we use the PSD of the individual pulse as a metric to evaluate alternative designs. In this paper we simulate a TH-PPM communication system for evaluating the performance of a signal designed using PSO. Table III gives the parameters utilized for the simulation of the communication system.

**TABLE III. UWB SIMULATION PARAMETERS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chip Width (ns)</td>
<td>(T_c)</td>
<td>2</td>
</tr>
<tr>
<td>Frame Width (ns)</td>
<td>(T_f)</td>
<td>32</td>
</tr>
<tr>
<td>Number of Chips Per Frame</td>
<td>(N_c)</td>
<td>16</td>
</tr>
<tr>
<td>Repetition Code Length</td>
<td>(N_s)</td>
<td>4</td>
</tr>
</tbody>
</table>

**V. RESULTS AND CONCLUSIONS**

We ran the PSO algorithm with 30 particles and for 1000 iterations. Multiple runs of algorithm converged to two competent designs. In the first two rows of Table V, the two composite pulses that occupy the full band, achieved using PSO are presented. The two pulses are also shown in Figure 6. The power values achieved by the two pulses are 362.89 \(\mu W\) and 369.44 \(\mu W\) respectively. The percentage of the achieved power to total power available within the spectrum (550 \(\mu W\)) is 65.9\% and 67.1\% respectively. Figure 4 shows the convergence of the PSO to a solution. The PSO algorithm converges to a solution within ~700- 800 iterations. The algorithm uses simple add and multiply operators to search through the space and arrive at solutions as presented in (9) and (10).

**A. Bit Error Rate Performance**

According to Shannon’s theory, the channel capacity of an Additive White Gaussian Noise (AWGN) channel is:
\[
C = B \log_2 (1 + SNR)
\]

The beauty of the new designed pulses is that, in theory, given a specific bandwidth \(B\) for the UWB signal, our designed pulse can provide more power than other pulses, which means we will have a better SNR value at the receiver, thus we can provide either higher data rate \(C\) or at the same \(C\) but with less BER at a fixed noise level. This is demonstrated in the bit error rate plot in Figure 5 as we compare the average BER performance for the PSO generated pulse and the optimized 5th derivative pulse presented in Table II. The simulations are carried out using a TH-PPM modulation scheme presented in the previous section. The parameters used for simulation are given in Table III. Since the two pulses designed used different power we normalize the signal to noise ratio (SNR) for fair comparisons. The average BER is plotted against normalized SNR in Figure 5. As we can see the composite pulse designed using PSO achieves lower average BER. The enhanced performance in terms of BER lets us do the trade-offs as per the Shannon’s theory.
The most important benefit of the proposed system lies in the ability to generate orthogonal pulses within the FCC mask. Using a bottom-up biologically inspired approach as in particle swarm optimization enables design of signals with arbitrary constraints. The speedy convergence and simplicity of the operators used by the algorithm enables its use in real time. An interesting feature of the design method is that since the pulses are linearly combined by different Gaussian derivative pulses, by using only a part of the pulses in the combination we can also design pulses that can be considered orthogonal due to their frequency separation. Figure 6 (Signal 1 and 2) shows two alternative designs that can be derived from the composite pulse design achieved by the PSO. Table IV gives the power values of this pulse in the two frequency bands. The signal C uses only the first derivative pulse from the composite pulse designed by the PSO. The pulse optimally occupies the 0-960 MHz band. The total power in this signal as given in Table V is 25.34 out of which 25.3324 is contained in the 0-960MHz band. The signal D only uses the 3, 4, 6 out of the second composite pulse presented in Table V. The pulse has most of the power concentrated in the second band i.e., 960MHz - 10.6 GHz and has only 6.5e-5 $\mu W$ which is hardly detectable and would not cause any interference with the first signal in case both are used simultaneously.

### Table IV. Orthogonality of Signals Generated by PSO in Frequency Domain

<table>
<thead>
<tr>
<th>Signal</th>
<th>0-960MHz</th>
<th>960MHz - 10.6GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>25.3324 $\mu W$</td>
<td>0.0076 $\mu W$</td>
</tr>
<tr>
<td>D</td>
<td>0.000065073 $\mu W$</td>
<td>344.10 $\mu W$</td>
</tr>
<tr>
<td>A</td>
<td>25.3324 $\mu W$</td>
<td>337.5576 $\mu W$</td>
</tr>
<tr>
<td>B</td>
<td>25.3324 $\mu W$</td>
<td>344.1076 $\mu W$</td>
</tr>
</tbody>
</table>

In this paper we presented a PSO algorithm to linearly combine the Gaussian derivatives creating multiple orthogonal UWB signals. These signals comply with the jagged FCC mask. The new composite pulse exploits the bandwidth and energy within a given spectral mask. The pulses generated by the PSO algorithm achieve 65% spectrum efficiency. In future work we intend to incorporate the number of orthogonal pulses as an objective function for evaluating the composite pulse in addition to the objective function (8). The algorithm will also be utilized to generate signals for sub-masks created with the FCC mask.

### Table V. Power/Parameter Values for Pulses Achieved Using Particle Swarm Optimization

<table>
<thead>
<tr>
<th>Signal</th>
<th>Derivatives Used</th>
<th>A(v)</th>
<th>$\alpha$ (ns)</th>
<th>Power $\mu W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1, 5, 6</td>
<td>0.80904, 11.3402, 4.350</td>
<td>1e-9*[1.969, 0.1809, 0.3070]</td>
<td>362.89</td>
</tr>
<tr>
<td>B</td>
<td>1, 3, 4, 6</td>
<td>0.80904, 5.3128, 14.2475, 6.6191</td>
<td>1e-9*[1.969, 0.1, 0.1635, 0.1]</td>
<td>369.44</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0.80904</td>
<td>1.969e-9</td>
<td>25.34</td>
</tr>
<tr>
<td>D</td>
<td>3, 4, 6</td>
<td>5.3128, 14.2475, 6.6191</td>
<td>1e-9*[0.1, 0.1635, 0.1]</td>
<td>344.10</td>
</tr>
</tbody>
</table>

### VI. REFERENCES


Figure 6. Multiple Signals Achieved using Particle Swarm Optimization Algorithm. The signals in time domain are presented in the first column and their equivalent frequency domain representation is presented on the right hand side.